Introduction to Lisp-Stat and its applications.

Sugested Solutions to Exercises 1.

1. ```
   (defun randu (n &optional (seed 12345))
        (let ((a (+ (^ 2 16) 3))
              (m (^ 2 31)))
          (flet ((mod (x m) (- x (* m (floor (/ x m))))))
             (do ((i 0 (1+ i))
                  (res (list seed)
                       (cons (mod (* a (first res)) m) res)))
                 ((>= i n)
                  (/ (rest (reverse res)) m))))))
   ```

   ```
   (def randu-lists
        (apply #'mapcar #'list (split-list (randu 300) 3)))
   ```

   ```
   (spin-plot randu-lists)
   ```

   Use the **Yaw** buttons to turn the plot. You may want to slow down the speed of the rotation using the **Slower** menu item a couple of times. In some views you will see 15 parallel lines. The generator will never fill all of $[0, 1]^3$ as it should if the observations are independent.

2. ```
   (load-data "book")
   (scatterplot-matrix (list flow moist ratio)
       :variable-labels
         (list "flow" "moist" "ratio"))
   (def hist1 (histogram flow))
   (send hist1 :variable-label 0 "flow")
   (send hist1 :adjust-to-data)
   (def hist2 (histogram flow))
   (send hist2 :variable-label 0 "moist")
   (send hist2 :adjust-to-data)
   (def hist3 (histogram flow))
   (send hist3 :variable-label 0 "ratio")
   (send hist3 :adjust-to-data)
   ```

   Link these plots and consider selecting various sets of observations.

3. (a)
```
(defun llik (theta)
  (let* ((x transformed-wbc-pos)
         (y times-pos)
         (n (length x))
         (theta0 (select theta 0))
         (theta1 (select theta 1)))
    (- (sum (* theta1 x))
       (* n (log theta0))
       (/ (sum (* y (exp (* theta1 x))))
          theta0))))
(def initial-vals
     (send (regression-model transformed-wbc-pos (log times-pos))
   :coef-estimates))

Least Squares Estimates:

Constant                    3.54234      (0.302699)
Variable 0                 -0.817801     (0.214047)

R Squared:                 0.493200
Sigma hat:                  1.23274
Number of cases:                 17
Degrees of freedom:              15
(setf (select initial-vals 0) (exp (select initial-vals 0)))
```

(b)
```
(newtonmax #'llik initial-vals)
maximizing...
Iteration 0.
Criterion value = -141.800
Iteration 1.
Criterion value = -103.559
Iteration 2.
Criterion value = -88.6502
Iteration 3.
Criterion value = -84.6240
Iteration 4.
Criterion value = -83.9533
```

```
Iteration 5.
Criterion value = -83.8792
Iteration 6.
Criterion value = -83.8774
Iteration 7.
Criterion value = -83.8774
Reason for termination: gradient size is less than gradient tolerance.
(56.84897833821361 0.48182887242189676)

(nelmeadmax #'llik initial-vals :verbose nil)
(56.85866918821114 0.4816337852905037)
```

I set the keyword argument `:verbose` to `nil` to avoid the printing of the function value at each of 61 iterations!

4. We could also conduct a Bayesian analysis of this data set examined in the previous question.

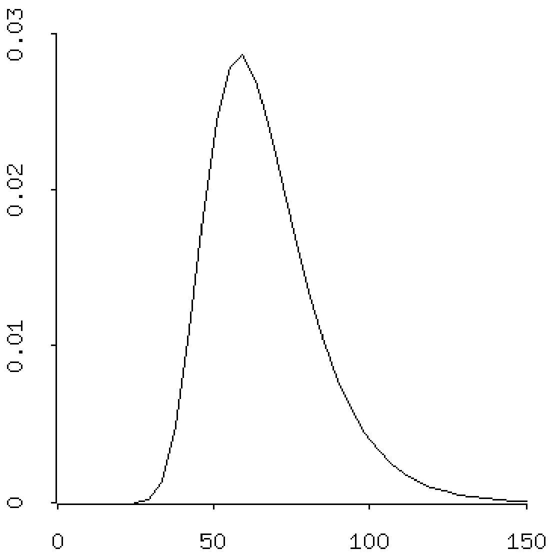
   (a)
```
(def leuk-bayes1 (bayes-model #'llik initial-vals))
maximizing...
Iteration 0.
Criterion value = -141.800
Iteration 1.
Criterion value = -103.559
Iteration 2.
Criterion value = -88.6502
Iteration 3.
Criterion value = -84.6240
Iteration 4.
Criterion value = -83.9533
Iteration 5.
Criterion value = -83.8792
Iteration 6.
Criterion value = -83.8774
Iteration 7.
Criterion value = -83.8774
Reason for termination: gradient size is less than gradient tolerance.
```

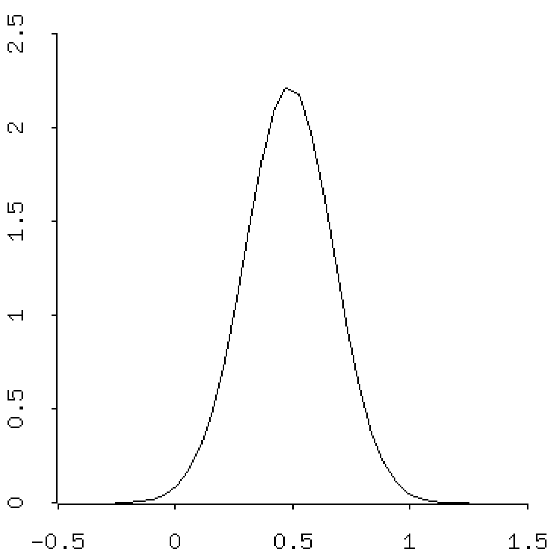
```
First Order Approximations to Posterior Moments:

Parameter 0                   56.8490      (13.9713)
Parameter 1                  0.481829      (0.179694)
```

(b)
```
(send leuk-bayes1 :moments)
((65.3209 0.4853) (17.1573 0.1866))
```

The Second order moment for θ_0 is larger than the first order suggesting that the mean is greater than the mode and so the posterior is positively skewed.

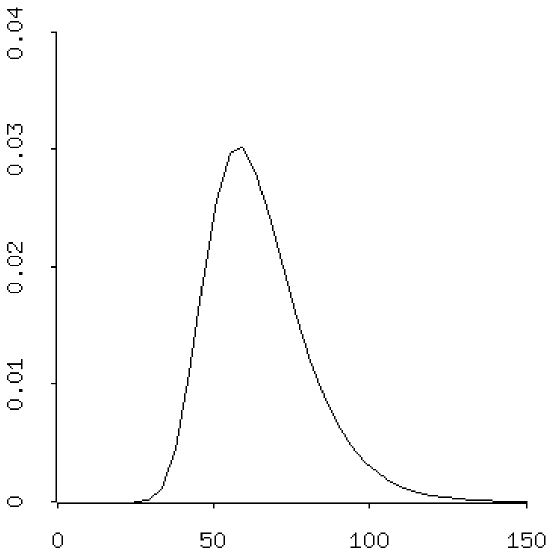
(c)
```
(plot-lines (send leuk-bayes1 :margin1 0 (rseq 25 150 100)))
(plot-lines (send leuk-bayes1 :margin1 1 (rseq -0.25 1.25 100)))
```

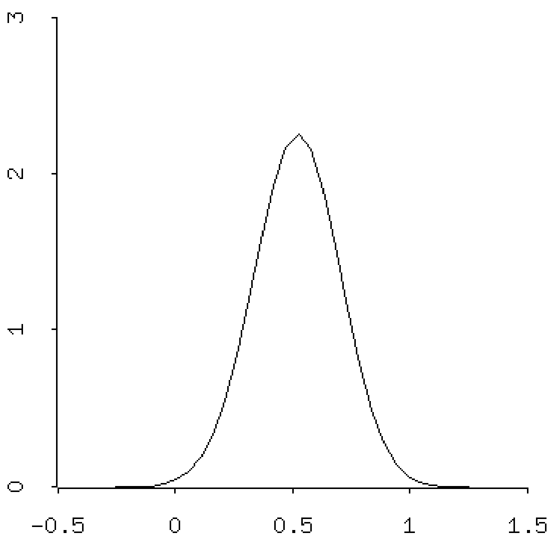




(d)
```
(defun prior0 (theta0)
   (normal-dens (/ (- (log theta0) (log 52)) (log 2))))
(defun prior1 (theta1)
   (normal-dens (/ (- theta1 5) 2)))
(defun lpost (theta)
   (+ (llik theta)
      (log (prior0 (first theta)))
      (log (prior1 (second theta)))))
(def leuk-bayes2 (bayes-model #'lpost initial-vals))
maximizing...
Iteration 0.
Criterion value = -148.043
Iteration 1.
Criterion value = -108.927
Iteration 2.
Criterion value = -93.3418
Iteration 3.
Criterion value = -89.0302
Iteration 4.
Criterion value = -88.3297
Iteration 5.
Criterion value = -88.2570
Iteration 6.
Criterion value = -88.2555
Iteration 7.
Criterion value = -88.2555
Reason for termination: gradient size is less than gradient tolerance.


First Order Approximations to Posterior Moments:

Parameter 0                 56.7388      (13.1752)
Parameter 1                0.516234      (0.176995)

(send leuk-bayes2 :moments)
((64.0463 0.5195) (15.7018 0.1822))
```

```
(plot-lines (send leuk-bayes2 :margin1 0 (rseq 25 150 100)))
(plot-lines (send leuk-bayes2 :margin1 1 (rseq -0.25 1.25 100)))
```





Again there is evidence of non-symmetry in the posterior for θ_0. The posteriors are not changed very much by using an informative prior. It is usually a good idea to look at different priors to assess the sensitivity of the posterior distribution to the priors. In this case the sensitivity is low suggesting that most information comes from the data rather than the priors.