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(A) Popular talk:
The geometry of random image in astrophysics and brain mapping

The geometry in the title is not the geometry of lines and angles but the geometry of topology, shape and knots. For example, galaxies are not distributed randomly in the universe, but they tend to form clusters, or sometimes strings, or even sheets of high galaxy density. How can this be handled statistically? The Euler characteristic (EC) of the set of high density regions has been used to measure the topology of such shapes; it counts the number of connected components of the set, minus the number of `holes,' plus the number of `hollows'. Despite its complex definition, the exact expectation of the EC can be found for some simple models, so that observed EC can be compared with expected EC to check the model. A similar problem arises in functional magnetic resonance imaging (fMRI), where the EC is used to detect local increases in brain activity due to an external stimulus. Recent work has extended these ideas to manifolds so that we can detect changes in brain shape via structure masking, surface extraction, and 3D deformation fields. Finally, we look at some curious random fields whose excursion sets are strings, and we show using the Siefert representation that these strings can be knotted.

(B) One long more technical talk:
Detecting changes in brain shape, scale and connectivity via the geometry of random fields

Three types of data are now available to test for changes in brain shape: 3D binary masks, 2D triangulated surfaces, and trivariate 3D vector displacement data from the non-linear deformations required to align the structure with an atlas standard. We use the Euler characteristic of the excursion set of a random field as a tool to test for localised shape changes. We extend these ideas to scale space, where the scale of the smoothing kernel is added as an extra dimension to the random field. Extending this further still, we look at fields of correlations between all pairs of voxels, which can be used to assess brain connectivity. Shape data is highly non-isotropic, that is, the effective smoothness is not constant across the image, so the usual random field theory does not apply. We propose a solution that warps the data to isotropy using local multidimensional scaling. We then show that the subsequent corrections to the random field theory can be done without actually doing the warping - a result guaranteed in part by the famous Nash Embedding Theorem. This has recently been formalized by Jonathan Taylor who has extended Robert Adler's random field theory to arbitrary manifolds.

(C) Theoretical talk:
Recent advances in random field theory

Since Robert Adler's 1981 book on the geometry of random fields, the many successful applications to astrophysics and brain mapping in the last 10 years have provoked a flurry of new theoretical work. We trace the history of this development over the last 20 years, touching on: Robert Adler's early work on the expected EC of excursion sets; David Siegmund's approach to finding the P-value of the maximum using Weyl's tube formula; Kuriki and Takemura's link between the two; Naiman and Wynn's improved Bonferroni inequalities; Robert Adler's proof that the expected EC really does approximate the P-value of the maximum; Jonathan Taylor's extensions to manifolds; the role of the Nash Embedding Theorem; and Jonathan Taylor's remarkable and unexpected Gaussian Kinematic Fundamental Formula for finding EC densities.