3me cycle de la Suisse Romande en Statistique, October, 2004

Inference and simulation for spatial point processes

Spatial point processes are used to model point patterns where the points typically are positions or centres of objects in a two- or three-dimensional region. The points may be decorated with marks (such as sizes or types of the objects) whereby marked point processes are obtained. The areas of applications are manifold and include astronomy, ecology, forestry, geography, image analysis, and spatial epidemiology. For more than 30 years spatial point processes have been a major area of research in spatial statistics. It is expected that research in spatial point processes will continue to be of importance as new technology makes huge amounts of spatial point process data available and new applications emerge.

Some of the earliest applications of computational methods in statistics are related to spatial point processes. In the last decade computational methods, and particularly Markov Chain Monte Carlo (MCMC) methods, have undergone major developments. These three talks (of 75-90 minutes each) are concerned with simulation-based inference for spatial point processes, with an emphasis on MCMC methods. Much of the material is covered by the monographs [3] and [6], and further references are given therein. My own publications can be found at `http://math.aau.dk/~jm`.

Naturally we cannot cover all topics within spatial point process theory, and the topics chosen are partly reflecting my own current research interests. For instance, an important topic such as summary statistics is not included. Further, Poisson, Cox and cluster processes are only briefly treated, while we focus on Markov point processes. Furthermore, a lot about perfect simulation for spatial point processes will be mentioned. Topic 3. below will only be considered if enough time is left.

1. Spatial point process models and simulation

- History and examples of spatial point patterns. Definition.
- Briefly about Poisson, Cox and cluster processes.
- Markov point processes:

  - Papangelou conditional intensity and local Markov property.
  - Hammersley-Clifford-Ripley-Kelly theorem and spatial Markov property.
  - Examples of Markov point processes.

- Metropolis-Hastings algorithms for spatial point processes with an unnormalised density:

  - Birth-death-move algorithm.
  - Local stability and convergence properties.

Some literature: [3] and [6].

2. Simulation-based inference and perfect simulation for spatial point process models with an unnormalised density

- Estimation of ratios of unknown normalising constants.

- Likelihood, pseudo-likelihood and Bayesian inference.

- An example: Bayesian analysis of deformed tessellation models.

- Short diversion into perfect simulation ideas:

  - The falling leaves model.
  - Widom-Rowlinson model.

- Dominated coupling from the past and spatial birth-death processes.

- Bayesian inference:

  - Parametric and semi-parametric Markov point process models.
  - An efficient Markov chain Monte Carlo method for distributions with intractable normalising constants.

Some literature: [1], [2], [3], [4] and [6].

3. Perfect simulation for Hawkes processes

Hawkes processes are Poisson cluster processes with certain branching and self-similarity properties. Such processes play a fundamental role for point process theory and its applications. Particularly, marked Hawkes processes have important applications in seismology and neurophysiology. We discuss how to make perfect simulations of (unmarked and marked) Hawkes processes.

Literature: [5].

References

[1] Blackwell, P. G. and Møller, J. (2003). Bayesian analysis of deformed tessellation models. *Advances in Applied Probability*, **35**, 4-26.

[2] Kendall, W. S. and Møller, J. (2000). Perfect simulation using dominating processes on ordered spaces, with application to locally stable point processes. *Advances in Applied Probability*, **32**, 844-865.

[3] Møller, J. (2003). *Spatial Statistics and Computational Methods.* Springer Lecture Notes in Statistics 173, Springer-Verlag, New York.

[4] Møller, J., Pettitt, A. N., Berthelsen, K. K. and Reeves, R. W. (2004). An efficient MCMC method for distributions with intractable normalising constants. Submitted. Available at `http://math.aau.dk/~jm`.

[5] Møller, J. and Rasmussen, J. G. (2004). Perfect simulation of Hawkes processes. Submitted. Available at `http://math.aau.dk/~jm`.

[6] Møller, J. and Waagepetersen, R. (2003). Statistical inference and simulation for spatial point processes. Chapman and Hall/CRC Press.