Filtering, smoothing and parameter estimation for general state space models

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Abstract

A general state space model (also called a hidden Markov model) consists of an unobservable Markovian state process (X_t) and a sequence of observations (Y_t) which are conditionally independent given the state variables, and such that Y_t depends only on X_t. These models have found a wide range of applications in engineering, environmental science, biology and financial mathematics. For all these applications, one has to solve at least some of the following problems: Inference about X_t given $Y_1, \ldots, Y_t$ (filtering), inference about $X_1, \ldots, X_t$ given $Y_1, \ldots, Y_t$ (smoothing) and estimation of unknown parameters in the model.

After presenting some examples of state space models, I will review the basic formulae for the filter and the smoothing distribution and for the likelihood. Explicit calculations are possible essentially only in two cases, namely discrete valued state variables and linear Gaussian models. Hence the main emphasis will be on Monte Carlo methods which are the focus of much ongoing research. In particular, sequential methods, also called particle filters, have been found to be a powerful tool. I will present the basic ideas plus some of the modifications that lead to more efficient algorithms.