(2,R), the Meixner classes and the renormalized square of white noise: an illustration of the serendipity of mathematics.

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In 1934 J.Meixner noticed a formal analogy among the generating functions of some famous classes of polynomials (Hermite, Charlier, Euler, ...) and found that there exist exactly five classes of probability measures with the property that the generating function of the associated orthogonal polynomials have precisely that structure. Thee are the: Gaussian, Poisson, Gamma, negative binomial (Pascal) and a new class, now called Meixner class.

Later on these measures were discovered to be infinitely divisible, but the main influence of Meixner's work was mainly confined to the theory of orthogonal polynomials and the probabilists payed little attention to it.

The situation begun to change at the end of the 1990's when, for different and independent motivations, these measures begun to appear in completely different fields of mathematics and statistics: the theory of unitary representations of groups and of the corresponding Lie algebras; the problem of optimal selection strategies based on relative ranks, when the total number of options is unknown; mathematical finance; the problem of chaotic and predictable representations for Lévy processes; the generalization of quantum fields (Jacobi fields); the theory of special functions and their relations with Lie algebras; the theory of interacting Fock spaces and quantum probability, ...

In 1999 Accardi, Lu and Volovich, studying the renormalized powers of free Boson fields (white noises) constructed the Fock representation for the square of white noise using techniques of interacting Fock spaces. One year later Accardi, Franz and Skeide proved that the abelian (vacuum) subprocesses of the renormalized square of white noise are in one-to-one correspondence with the Levy processes associated to the non standard (i.e. non Gaussian or Poisson) Meixner classes.

At the light of these results the formal similarity noticed by Meixner in 1934 acquires a deeper mathematical meaning: the first two Meixner classes are precisely the abelian subprocesses of the first order white noise, while the remaining three classes correspond to the renormalized second power.

The conceptual implications of this unification will be discussed at the light of the new idea of "quantum decomposition of a classicl random variable".