

Lectures at Champéry, 3ème cycle, March 3-6, 2002

## Empirical Processes in Statistics: Methods, Examples, Further Problems

by Jon A. Wellner

**Abstract:** Suppose that  $X_1, \dots, X_n$  are i.i.d. according to a distribution  $P$  on a measurable space  $(\mathcal{X}, \mathcal{A})$ . Modern empirical process theory concerns itself with inequalities and convergence results for the empirical measure  $\mathbb{P}_n = n^{-1} \sum_{i=1}^n \delta_{X_i}$  and the empirical process

$$\mathbb{G}_n = \sqrt{n}(\mathbb{P}_n - P)$$

considered as processes indexed by a class  $\mathcal{F}$  of real-valued functions from  $\mathcal{X}$  to  $R$ . In particular the theory has developed a large number of inequalities and tools for dealing with suprema such as

$$\sup_{f \in \mathcal{F}} |\mathbb{P}_n(f) - P(f)| \equiv \|\mathbb{P}_n - P\|_{\mathcal{F}}.$$

In these three lectures, I will first review a selection of some basic inequalities and convergence theorems for empirical processes. Then I will illustrate the application of some of these results to several examples in statistics. The lectures will conclude with a brief discussion of a few of the many open problems.

### Tentative Lecture Titles and Topics:

#### L1. Examples and Empirical Process Basics:

- Some Examples.
  - (a) Several simple 1-d examples
  - (b) Interval censoring problems in 1 and higher dimensions
  - (c) Mixture models and profile likelihood
- Notation and history
- Useful inequalities
- Converg numbers and bracketing numbers
- Some basic convergence theorems
- A preview: connecting with statistics

**L2.** Empirical Process Methods for statistics:

- Armax-continuous mapping theorem.
- M-estimates and Z-estimates: rates of Convergence.
- M-estimates and Z-estimates: limiting distributions.
- Back to the examples.

**L3.** Extensions and Further Problems

**Some References:**

- van der Vaart, A. W. and Wellner, J. A. (1996). *Weak Convergence and Empirical Processes*. Springer-Verlag, New York.
- van der Vaart, A. W. and Wellner, J. A. (2000). Preservation theorems for Glivenko-Cantelli and uniform Glivenko-Cantelli classes, pp. 115 - 134 In *High Dimensional Probability II*, Birkhäuser, Boston.
- van der Vaart, A. W. (2000). Semiparametric Statistics. Lectures on Probability Theory, Ecole d'Ete de Probabilites de St. Flour-XX, 1999. P. Bernard, Ed. Springer, Berlin. To appear.
- Pollard, D. (1990). *Empirical Processes: Theory and Applications*. NSF-CBMS Regional Conference Series in Probability and Statistics **2**, Institute of Mathematical Statistics.