Wavelet Analysis of Discrete Time Series
Lecturers: Don Percival & Andrew Walden

Just as a square integrable signal can be represented by weighted sines and cosines in Fourier analysis, so can it alternatively be represented in terms of weighted small 'waves' or wavelets. These wavelets are localised in time and scale, enabling the detection of sudden changes or patterns of change at particular scales. A plot of these weights in time/scale gives a visual picture of the continuous wavelet transform (CWT). The discrete wavelet transform (DWT) is essentially a subsampling of the CWT at just the dyadic scales 2^{j-1}, $j = 1, 2, \ldots$, and can be very rapidly computed using a 'pyramid algorithm.' This series of lectures discusses the structure of the DWT and some important variants, and the application of these transforms to the statistical analysis of discrete time series. Substantive examples of applications will be presented.

- Lecture 1 (Sunday, 8 September, 16:30) [Percival] Introduction. The essence of wavelet analysis. The CWT and the interpretation of scale. The DWT as a subsampling of the CWT. Preserving some of the CWT's redundancy — the maximal overlap DWT (MODWT).
- Lecture 2 (Monday, 9 September, 08:30) [Walden] Algorithms and properties. Filter design. Iterative filtering and downsampling: the pyramid algorithm for the DWT. The DWT by direct filtering of the data. Phase of the filters. Elimination of downsampling: the MODWT. Increasing frequency resolution via the discrete wavelet packet transform (DWPT) and its non-downsampled version (MODWPT). Examples.
- Lecture 3 (Monday, 9 September, 10:30) [Percival/Walden] The wavelet variance: definition, rationale, properties, estimation and confidence intervals. Application to non-Gaussian time series. Examples. Extensions to wavelet covariance and cross-covariance and further statistical results for non-Gaussian time series. Examples.
- Lecture 4 (Monday, 9 September, 16:30) [Percival] Analysis and synthesis of a long-memory process via the DWT. Maximum likelihood estimation for fractionally-differenced processes (stationary and nonstationary). Least squares estimation. Testing for homogeneity of variance. Examples.
- Lecture 5 (Tuesday, 10 September, 08:30) [Walden] Signal estimation and denoising. DWT-based wavelet thresholding and shrinkage for IID Gaussian noise. Spectrum estimation via wavelet thresholding: uncorrelated non-Gaussian noise versus correlated Gaussian noise. Comparison with a DWPT approach. Examples.
- Lecture 6 (Tuesday, 10 September, 10:30) [Percival/Walden] Emerging techniques. Adaptive wavelet-based bootstrapping. Examples. Wavelet analysis of matrix-valued time series. Example.