LECTURE 1

SELF-SIMILARITY AND COMPUTER NETWORK TRAFFIC

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In this lecture we will introduce self-similarity in the context of computer network traffic. It will show why self-similarity is important in this area and will motivate the subsequent lectures.

Ethernet local area network traffic appears to be approximately statistically self-similar. This discovery, made about eight years ago, has had a profound impact on the field. I will try to explain what statistical self-similarity means, how it is detected and indicate how one can construct random processes with that property by aggregating a large number of "on-off" renewal processes. If the number of replications grows to infinity then, after rescaling, the limit turns out to be the Gaussian self-similar process called fractional Brownian motion. If, however, the rewards are heavy-tailed as well, then the limit is a stable non-Gaussian process with infinite variance and dependent increments. Since linear fractional stable motion is the stable counterpart of the Gaussian fractional Brownian motion, a natural conjecture is that the limit process is linear fractional stable motion. This conjecture, it turns out, is false. The limit is a new type of infinite variance self-similar process.

References

M. S. Taqqu, W. Willinger, R. Sherman "
"On-Off Models for Generating Long-Range Dependence"
Computer Communication Review, Vol 27 (1997), 5-23.

LECTURE 2

FRACTIONAL BROWNIAN MOTION, LONG-RANGE DEPENDENCE AND FARIMA MODELS

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Long-range dependence in a stationary time series occurs when the covariances tend to zero like a power function and so slowly that their sums diverge. It is often observed in nature, for example in economics, telecommunications and hydrology. It is closely related to self-similarity. Self-similarity refers to invariance in distribution under a suitable change of scale. To understand the relationship between self-similarity and long-range dependence, suppose that the self-similar process has stationary increments. Then these increments form a stationary time series which can display long-range dependence. Conversely, start with a stationary time series (with long-range dependence). Then a central limit-type theorem will yield a self-similar process with stationary increments. The intensity of long-range dependence is related to the scaling exponent of the self-similar process.

We shall provide here a tutorial on fractional Brownian motion, the Gaussian self-similar process with stationary increments, on its increment process known as fractional Gaussian noise, which displays long-range dependence, and on a large class of long-range dependent stationary sequences called FARIMA, which are commonly used in modeling such physical phenomena.

Ref:

M. S. Taqqu
"Fractional Brownian motion and long-range dependence". Preprint 2000.

LECTURE 3

SELF-SIMILARITY AND LONG-RANGE DEPENDENCE THROUGH THE WAVELET LENS

We provide a brief introduction to wavelets and describe how self-similar and long-range dependent processes can be detected by using the discrete wavelet transform. We discuss the nature of the wavelet coefficients and their statistical properties. The Logscale Diagram is introduced as a natural means to study scaling data and we show how it can be used to obtain unbiased semi-parametric estimates of the scaling exponent. We then focus on the case of long-range dependence and address the problem of defining a lower cutoff scale corresponding to where scaling starts. We also discuss some related problems arising from the application of wavelet analysis to discrete time series. Numerical examples using many discrete time models are then presented to show the quality of the wavelet-based estimator and how it compares with alternative ones. The examples include strong short range dependence, and non-Gaussian series with both finite and infinite variance.

Ref:

P. Abry, P. Flandrin, M. S. Taqqu and D. Veitch
"Self-similarity and long-range dependence through the wavelet lens". Preprint 2000.