Modelling Financial Time Series

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The aim of this short course is to consider some of the standard models for financial log-returns and to study their probabilistic properties, in particular their dependence structure and the behaviour of their extremes.

We start by discovering some of the "stylized facts" from real-life log-return data: long-range dependence effects which affect the sample autocorrelation behaviour of the absolute log-returns, bursty and peaky behaviour which results in heavy-tailed marginal distributions. In practice one often observes that financial log-return series can have infinite 3rd, 4th, 5th,... moments.

Then we look at some of the standard econometric models which try to capture the empirical behaviour. Those include GARCH (generalized autoregressive conditionally heteroscedastic) and SV (stochastic volatility) processes. We study their probabilistic structures. The GARCH case turns out to be a very complex one. We embed these processes in finite-dimensional stochastic recurrence equations. Following classical work of Kesten (1973), we show that the finite-dimensional distributions of these processes are multivariate regularly varying, thus have infinite power moments of a certain degree. Heavy tails cause the classical limit theory for the sample autocorrelations to break down. Thus one has to modify this asymptotic theory: infinite variance limit distributions occur and rates of convergence can be extremely slow. Point process techniques turn out to be important in this context.

The long-range dependence effect of real-life data cannot be explained by the models mentioned so far. A possible explanation for this effect is non-stationarity of the underlying time series. We will learn how statistical tools behave under non-stationarity and how they can fool us to see things which are not there.

Finally, we point out how the tails of such processes can be estimated by statistical means. In financial practice, tail estimation has become an important tool since regulators of banks require to estimate the 5%- or 1%-quantiles (Value at Risk -VaR) of profit-loss distributions of risky assets to measure the risk of such assets.

Literature

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